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Supersymmetric radion in the Randall-Sundrum scenario

Jonathan Bagger

Department of Physics and Astronomy, The Johns Hopkins University
3400 North Charles Street, Baltimore, MD 21218, USA
E-mail: bagger@jhu.edu

Dennis Nemeschansky

CIT-USC Center for Theoretical Physics and Dept. of Physics and Astronomy
University of Southern California, Los Angeles, CA 90089, USA
E-mail: dennisn@physics.usc.edu

Ren-Jie Zhang

School of Natural Sciences, Institute for Advanced Study
Olden Lane, Princeton, NJ 08540, USA
E-mail: rjzhang@ias.edu

Abstract: We derive the effective action for the radion supermultiplet in the supersymmetric Randall-Sundrum model with two opposite tension branes.

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1. Introduction

The two-brane Randall-Sundrum scenario [1] has attracted considerable attention because it generates the electroweak hierarchy as a consequence of spacetime geometry. The simplest version contains two opposite tension 3-branes placed at the boundaries of a slice through 4+1 dimensional anti de Sitter space. The hierarchy is determined by the brane tensions and by the distance between the branes. The distance is set by the expectation value of a modulus field, called the radion. Supersymmetry provides a natural mechanism for stabilizing the radion against radiative corrections.

During the past year, supersymmetric versions of the two-brane Randall-Sundrum scenario were constructed [2, 3]. Furthermore, the corresponding low-energy effective supergravity action was derived — for the case of fixed radion. In this paper we will find the effective field theory for the field radion itself. We will derive the $N = 1$ matter-coupled supergravity theory that describes the low energy dynamics of the radion.

The plan of this paper is as follows. In section 2 we review the five dimensional bulk-plus-brane supersymmetric action. In section 3 we derive the bosonic part of the low energy effective action. We will see that the Kaluza-Klein reduction requires a careful treatment of tadpoles associated with the massive fields. In section 4 we construct the fermionic part of the effective action, and in section 5 we derive the supersymmetry transformations for the zero mode fields. We will find that the lagrangian and transformation laws correspond to the usual $N = 1$ matter-coupled supergravity, with a Kähler potential of a particular form. We conclude in section 6 with a simple argument that produces the same Kähler potential.

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2. The five dimensional supersymmetric action

Our starting point is the supersymmetric Randall-Sundrum model presented in [2]. The total action contains bulk and brane pieces,

$$S = S_{\text{bulk}} + S_{\text{brane}}\,. \tag{2.1}$$

The bulk action is that of five dimensional supergravity [5], while S_{brane} arises from the presence of two opposite-tension branes.[1]

The bulk action is given by

$$\begin{aligned} S_{\text{bulk}} = \Lambda \int d^5x e \Bigg[& -\frac{1}{2\kappa^2} R + \mathrm{i}\epsilon^{MNOPQ} \overline{\Psi}_M \Sigma_{NO} D_P \Psi_Q - \frac{1}{4} F_{MN} F^{MN} - \\ & - 3\Lambda \bar{\Psi}_M \Sigma^{MN} \Psi_N + 6\frac{\Lambda^2}{\kappa^2} - \mathrm{i}\kappa \sqrt{\frac{3}{2}} \frac{1}{2} F_{MN} \bar{\Psi}^M \Psi^N - \\ & - \kappa \frac{1}{6\sqrt{6}} \epsilon^{MNOPQ} F_{MN} F_{OP} B_Q + \mathrm{i}\kappa \sqrt{\frac{3}{2}} \frac{1}{4} \epsilon^{MNOPQ} F_{MN} \bar{\Psi}_O \Gamma_P \Psi_Q - \\ & - \kappa\Lambda \sqrt{\frac{3}{2}} \epsilon^{MNOPQ} \bar{\Psi}_M \Sigma_{OP} \Psi_N B_Q + \text{ four-Fermi terms } \Bigg]\,. \end{aligned} \tag{2.2}$$

In this expression, the parameter κ is related to the effective four dimensional Planck constant, $\kappa^2 = \bar{\kappa}^2(1 - e^{-2\pi r\Lambda})$, and the ϵ tensor is defined to have tangent-space indices,[2] with $\epsilon^{01235} = 1$. The action contains the physical fields of the supergravity multiplet in five dimensions: the fünfbein $e_M{}^A$, the gravitino Ψ_M, and the graviphoton B_M. The coordinate $x^5 = z = r\phi$ parameterizes the orbifold S^1/Z_2, where the circle S^1 has radius r and the orbifold identification is $\phi \leftrightarrow -\phi$. We choose to work on the orbifold covering space, so we take $-\pi < \phi \leq \pi$. The orbifold breaks $N = 1$ supersymmetry in five dimensions to $N = 1$ in four.

The brane action is intrinsically four dimensional, so we write the five dimensional spinors in four dimensional notation, where

$$\Psi_M = \begin{pmatrix} \psi^1_{M\alpha} \\ \bar{\psi}^{\dot{\alpha}}_{2M} \end{pmatrix} \tag{2.3}$$

and

$$\Gamma^a = \begin{pmatrix} 0 & \sigma^a_{\alpha\dot{\alpha}} \\ \bar{\sigma}^{a\dot{\alpha}\alpha} & 0 \end{pmatrix} \qquad \Gamma^{\hat{5}} = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix}. \tag{2.4}$$

The symbol $\hat{5}$ denotes the fifth tangent space index; the fields ψ^i_M are two-component Weyl spinors. We define $\psi^\pm_M = \frac{1}{\sqrt{2}}(\psi^1_M \pm \psi^2_M)$, and likewise for the supersymmetry transformation parameter $\eta^\pm$.

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[1] Our final results are independent of whether we start with the five dimensional action presented in ref. [2] or [3].

[2] We adopt the convention that capital letters run over the set $\{0, 1, 2, 3, 5\}$ and lower-case letters run from 0 to 3. Tangent space indices are taken from the beginning of the alphabet; coordinate indices are from the middle. We follow the conventions of [2, 4].

In this language, the brane action is simply

$$S_{\text{brane}} = \frac{\Lambda}{\kappa^2} \int d^5x \hat{e}(-3\Lambda + 2\kappa^2 \psi_m^+ \sigma^{mn} \psi_n^+) \left[\delta(z) - \delta(z - \pi r)\right] + \text{h.c.}\,, \tag{2.5}$$

where $\hat{e} = \det e_m{}^a$ and the $e_m{}^a$ are the components of the fünfbein, restricted to the appropriate brane.

The full bulk plus brane action is invariant under the following transformations,

$$\begin{aligned}
\delta e_M{}^a &= \mathrm{i}\kappa(\eta^+\sigma^a\bar{\psi}_M^+ + \eta^-\sigma^a\bar{\psi}_M^-) + \text{h.c.} \\
\delta e_M{}^{\hat{5}} &= \kappa(\eta^+\psi_M^- - \eta^-\psi_M^+) + \text{h.c.} \\
\delta B_M &= -\mathrm{i}\sqrt{\frac{3}{2}}(\eta^+\psi_M^- - \eta^-\psi_M^+) + \text{h.c.} \\
\delta\psi_m^\pm &= \frac{2}{\kappa}D_m\eta^\pm \mp \frac{\mathrm{i}}{\kappa}\omega_{ma\hat{5}}\sigma^a\bar{\eta}^\mp \pm \mathrm{i}\frac{\Lambda}{\kappa}e_m{}^a\sigma_a\bar{\eta}^\pm + e_{m\hat{5}}\frac{\Lambda}{\kappa}\eta^\mp - \mathrm{i}\sqrt{6}\Lambda B_m\eta^\mp \; - \\
&\quad - \sqrt{\frac{2}{3}}\Bigg(\mp e_a{}^N F_{Nm}\sigma^a\bar{\eta}^\mp - \mathrm{i}e_{\hat{5}}{}^N F_{Nm}\eta^\pm - \frac{1}{4}\epsilon_{ABCde}e_m{}^A e^{BN} e^{CO} F_{NO}\sigma^{de}\eta^\pm \; \pm \\
&\qquad \pm \frac{\mathrm{i}}{4}\epsilon_{abcd}e_m{}^a e^{bN} e^{cO} F_{NO}\sigma^d\bar{\eta}^\mp\Bigg) \\
\delta\psi_5^+ &= \frac{2}{\kappa}D_5\eta^+ - \frac{\mathrm{i}}{\kappa}\omega_{5a\hat{5}}\sigma^a\bar{\eta}^- + e_{5\hat{5}}\frac{\Lambda}{\kappa}\eta^- + \mathrm{i}e_5{}^a\frac{\Lambda}{\kappa}\sigma_a\bar{\eta}^+ - \mathrm{i}\sqrt{6}\Lambda B_5\eta^- \; - \\
&\quad - \sqrt{\frac{2}{3}}\Bigg(e_a{}^n F_{5n}\sigma^a\bar{\eta}^- + \mathrm{i}e_{\hat{5}}{}^n F_{5n}\eta^+ - \frac{1}{4}\epsilon_{ABCde}e_5{}^A e^{BN} e^{CO} F_{NO}\sigma^{de}\eta^+ \; - \\
&\qquad - \frac{\mathrm{i}}{4}\epsilon_{abcd}e_5{}^a e^{bN} e^{cO} F_{NO}\sigma^d\bar{\eta}^-\Bigg) \\
\delta\psi_5^- &= \frac{2}{\kappa}D_5\eta^- + \frac{\mathrm{i}}{\kappa}\omega_{5a\hat{5}}\sigma^a\bar{\eta}^- + e_{5\hat{5}}\frac{\Lambda}{\kappa}\eta^+ - \mathrm{i}e_5{}^a\frac{\Lambda}{\kappa}\sigma_a\bar{\eta}^- - \mathrm{i}\sqrt{6}\Lambda B_5\eta^+ \; + \\
&\quad + \sqrt{\frac{2}{3}}\Bigg(e_a{}^n F_{5n}\sigma^a\bar{\eta}^+ - \mathrm{i}e_{\hat{5}}{}^n F_{5n}\eta^- + \frac{1}{4}\epsilon_{ABCde}e_5{}^A e^{BN} e^{CO} F_{NO}\sigma^{de}\eta^- \; - \\
&\qquad - \frac{\mathrm{i}}{4}\epsilon_{abcd}e_5{}^a e^{bN} e^{cO} F_{NO}\sigma^d\bar{\eta}^+\Bigg) \; - \\
&\quad - \frac{4}{\kappa}[\delta(z) - \delta(z - \pi r)]\eta^+\,,
\end{aligned} \tag{2.6}$$

provided the fields and transformation parameters obey the appropriate jump conditions at the locations of the branes. In these expressions, all covariant derivatives contain the spin connection ω_{Mab}. Here and hereafter, we ignore all three- and four-Fermi terms.

The fields and transformation parameters are assigned definite Z_2 parities under the orbifold symmetry $\phi \to -\phi$. We choose

$$e_m{}^a\,, \qquad e_{5\hat{5}}\,, \qquad B_5\,, \qquad \psi_m^+\,, \qquad \psi_5^-\,, \qquad \eta^+$$

to have even parity, and

$$e_5{}^a\,, \qquad e_m{}^{\hat 5}\,, \qquad B_m\,, \qquad \psi_m^-\,, \qquad \psi_5^+\,, \qquad \eta^-$$

to have odd. These parity assignments are consistent with the action and the supersymmetry transformations.

3. The bosonic effective action

In this paper we derive the effective field theory for the zero modes associated with the five dimensional supergravity fields. We consider the four dimensional $N = 1$ supergravity multiplet, as well as the four dimensional chiral multiplet that contains the radion. In this section we find the bosonic part of the low energy effective action.

We take the vacuum to be the original Randall-Sundrum solution [1],

$$ds^2 = e^{-2\sigma}\eta_{mn}dx^m dx^n + dz^2\,. \tag{3.1}$$

This solves the five dimensional Einstein equations

$$R_{MN} - \frac{1}{2}g_{MN}R = -6g_{MN}\Lambda^2 + 6g_{mn}\delta^m_M\delta^n_N\Lambda\left(\frac{\hat e}{e}\right)[\delta(z) - \delta(z-\pi r)] \tag{3.2}$$

when η_{mn} is the flat Minkowski metric and $\sigma = \Lambda|z|$.

The low energy effective action describes the light fields that fluctuate off this background. The effective theory of the gravitational field was derived in ref. [1]. The zero mode metric is

$$ds^2 = e^{-2\sigma}\bar g_{mn}dx^m dx^n + dz^2\,, \tag{3.3}$$

where $\bar g_{mn}$ is the effective four dimensional metric, a function of $x^0,\ldots,x^3$, but not x^5. The effective action follows from substituting (3.3) into (2.1) and integrating over x^5. One finds

$$S_{\text{eff}} = -\frac{1}{2\bar\kappa^2}\int d^4x\bar e\bar R, \tag{3.4}$$

where $\bar R$ is the four dimensional Ricci scalar constructed from the metric $\bar g$. This is nothing but the Einstein action in four dimensions, with an effective four dimensional squared Planck mass $\bar\kappa^{-2} = \kappa^{-2}(1 - e^{-2\pi r\Lambda})$.

This procedure is consistent because the five dimensional Einstein equations (3.2) are satisfied when

$$\bar R_{mn} = 0\,. \tag{3.5}$$

Indeed, the ansatz (3.3) gives a solution, point-by-point in x^5, precisely when $\bar g_{mn}$ satisfies the four dimensional Einstein equations, derived from the effective action (3.4). This implies that the Kaluza-Klein reduction is consistent; all higher Kaluza-Klein modes can be set to zero.

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In what follows we generalize this reduction to include the radion field, A. We construct a four dimensional effective action that is correct to all orders for constant radion A, and correct up to two derivatives when A depends on the four dimensional coordinates $x^0, \ldots, x^3$. The resulting effective action describes the leading low energy dynamics of the radion field.

We start by writing a five dimensional metric that satisfies the Einstein equations for constant A. Such a metric is given by

$$ds^2 = e^{-2F(A,\sigma)} \bar{g}_{mn} dx^m dx^n + \left| \frac{\partial F(A,\sigma)}{\partial \sigma} \right|^2 dz^2 \,. \tag{3.6}$$

For constant A, (3.6) can be obtained from (3.3) by a coordinate transformation, so it automatically satisfies the five dimensional equations of motion.

When the field A depends on the four dimensional coordinates $x^0, \ldots, x^3$, the metric (3.6) is not a coordinate transformation of (3.3), and the reduction is more complicated. The problem is that A mixes with the graviton $\bar{g}_{mn}$ and with its Kaluza-Klein excitations $\bar{h}_{mn}$. Moreover, the amount of mixing typically depends on the coordinate x^5. This makes it subtle to extract the four dimensional effective theory.

These problems are eliminated by choosing the following ansatz for the five dimensional metric [3]

$$ds^2 = e^{-2\sigma} \bar{g}_{mn} (1 + A e^{2\sigma}) dx^m dx^n + \frac{1}{(1 + A e^{2\sigma})^2} dz^2 \,. \tag{3.7}$$

This metric is of the form (3.6), where

$$F(A, \sigma) = \sigma - \frac{1}{2} \log(1 + A e^{2\sigma}) \,. \tag{3.8}$$

With this ansatz, the gravitational part of the five dimensional action (2.1) is just

$$S = \Lambda \int d^5x \bar{e} \left[-\frac{1}{2\kappa^2} e^{-2\sigma} \bar{R} - \frac{3}{4\kappa^2} \frac{e^{2\sigma}}{(1 + A e^{2\sigma})^2} \bar{g}^{mn} \partial_m A \partial_n A \right] . \tag{3.9}$$

Equation (3.9) is in the "Einstein frame" at each point in x^5. This guarantees that the radion never mixes with the graviton, to any order in the fields.

The radion effective action follows from integrating (3.9) over x^5. This gives

$$\begin{aligned} S_{\text{eff}} &= \int d^4x \bar{e} \left[-\frac{1}{2\bar{\kappa}^2} \bar{R} - \frac{3}{4\bar{\kappa}^2} \frac{e^{2\pi r \Lambda}}{(1 + A)(1 + A e^{2\pi r \Lambda})} \bar{g}^{mn} \partial_m A \partial_n A \right] \\ &= \int d^4x \bar{e} \left[-\frac{1}{2\bar{\kappa}^2} \bar{R} - \frac{1}{8\bar{\kappa}^2} \sinh^{-2} \left(\frac{B}{\sqrt{6}} \right) \bar{g}^{mn} \partial_m B \partial_n B \right] , \end{aligned} \tag{3.10}$$

[3]This metric coincides, to linear order, with the metric proposed in [6].

where we write A in terms of the x^5-invariant proper length, B,

$$\sqrt{\frac{2}{3}}B = 2\Lambda \int_0^{\pi r} e_{5\hat{5}} dz = 2\pi r\Lambda + \log\left[\frac{1+A}{1+Ae^{2\pi r\Lambda}}\right]. \tag{3.11}$$

The field B describes the physical distance between the opposite-tension branes.

Let us now examine the consistency of this reduction. The four dimensional equations of motion are easily derived from (3.10). We find

$$\bar{R}_{mn} = -\frac{1}{4}\sinh^{-2}\left(\frac{B}{\sqrt{6}}\right)\partial_m B\partial_n B \tag{3.12}$$

and

$$\Box B - \frac{1}{\sqrt{6}}\coth\left(\frac{B}{\sqrt{6}}\right)\bar{g}^{mn}\partial_m B\partial_n B = 0\,. \tag{3.13}$$

The five dimensional equations are obtained by substituting the metric (3.7) into the equations (3.2). The (mn) and (55) Einstein equations reduce to

$$\bar{R}_{mn} = -\frac{3}{2}\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2 \partial_m A\partial_n A \tag{3.14}$$

and

$$\Box A = \frac{e^{2\sigma}}{1+Ae^{2\sigma}}\bar{g}^{mn}\partial_m A\partial_n A\,. \tag{3.15}$$

The $(m5)$ equation is satisfied for any value of the radion field.

Equations (3.14) and (3.15) show that the Kaluza-Klein reduction is *not* consistent as it stands. The fields $\bar{R}$ and A depend only on $x^0,\ldots,x^3$, so eqs. (3.14) and (3.15) cannot be satisfied point by point in x^5. In fact, they must be averaged over the fifth dimension. This gives

$$\begin{aligned}
\bar{R}_{mn} &= 2\Lambda\left(\frac{\bar{\kappa}}{\kappa}\right)^2 \int_0^{\pi r} dz e^{-2\sigma}\bar{R}_{mn} \\
&= -\frac{3}{2}\frac{e^{2\pi r\Lambda}}{(1+A)(1+Ae^{2\pi r\Lambda})}\partial_m A\partial_n A \\
&= -\frac{1}{4}\sinh^{-2}\left(\frac{B}{\sqrt{6}}\right)\partial_m B\partial_n B
\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}
0 &= 2\Lambda\left(\frac{\bar{\kappa}}{\kappa}\right)^2 \int_0^{\pi r} dz e^{-2\sigma}\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2\left(\Box A - \frac{e^{2\sigma}}{1+Ae^{2\sigma}}\bar{g}^{mn}\partial_m A\partial_n A\right) \\
&= -\sqrt{\frac{3}{2}}\left[\Box B - \frac{1}{\sqrt{6}}\coth\left(\frac{B}{\sqrt{6}}\right)\bar{g}^{mn}\partial_m B\partial_n B\right],
\end{aligned} \tag{3.17}$$

in which case they reduce to (3.12) and (3.13), respectively.

This apparent inconsistency has an important physical origin. It stems from the fact that the higher Kaluza-Klein modes cannot be set to zero. Their equations of motion are such that $\bar{h}_{mn}$ and $\bar{h}_{55}$ acquire tadpoles proportional to derivatives of the light fields,

$$\bar{h}_{mn} \sim \partial_m B \partial_n B \,, \qquad \bar{h}_{55} \sim \bar{g}^{mn} \partial_m B \partial_n B \,. \tag{3.18}$$

These tadpoles alter eqs. (3.14) and (3.15) and restore the consistency of the Kaluza-Klein reduction.

In general, these tadpoles also change the low energy effective action. In this paper, however, we work to *second order* in spacetime derivatives. To this order, the $\bar{h}_{mn}$ and $\bar{h}_{55}$ tadpoles can be neglected. The $\bar{h}_{m5}$ field can induce the only dangerous terms. However, for our ansatz, the $(m5)$ Einstein equation is always satisfied, so $\bar{h}_{m5}$ does not acquire a tadpole. Therefore (3.10) is indeed the consistent low energy effective action.

In the rest of this section, we find the Kaluza-Klein reduction for the graviphoton, B_M. As above, we start with the five dimensional equation of motion,

$$\partial_M \left(e g^{MN} g^{PQ} F_{NQ} \right) = 0 \,. \tag{3.19}$$

In the radion background (3.7), this reduces to

$$\begin{aligned} 0 &= \partial_m \left[\bar{e} e^{-2\sigma} (1 + A e^{2\sigma})^2 \bar{g}^{mn} F_{n5} \right] \\ 0 &= \partial_5 \left[\bar{e} e^{-2\sigma} (1 + A e^{2\sigma})^2 \bar{g}^{pq} F_{5q} \right] + \partial_m \left[\bar{e} (1 + A e^{2\sigma})^{-1} \bar{g}^{mn} \bar{g}^{pq} F_{nq} \right] . \end{aligned} \tag{3.20}$$

We first find a solution for constant radion. We take

$$B_m = 0 \,, \qquad B_5 = \frac{\alpha e^{2\sigma}}{\kappa (1 + A e^{2\sigma})^2} C \,, \tag{3.21}$$

where

$$\alpha = \frac{(1 + A)(1 + A e^{2\pi r \Lambda})}{(e^{2\pi r \Lambda} - 1)} \tag{3.22}$$

and C is the gauge-invariant Aharonov-Bohm phase around the fifth dimension,

$$C = 2\kappa\Lambda \int_0^{\pi r} dz B_5 \,. \tag{3.23}$$

From these expressions it is easy to calculate the field strengths. We find

$$F_{m5} = \frac{\alpha e^{2\sigma}}{\kappa (1 + A e^{2\sigma})^2} \partial_m C \tag{3.24}$$

and

$$F_{mn} = 0 \,. \tag{3.25}$$

For constant A, eqs. (3.24) and (3.25) satisfy the five dimensional equations of motion, provided C is a massless scalar field, satisfying $\Box C = 0$.

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Let us now fluctuate the radion field. We assume, for the moment, that the curvatures (3.24) and (3.25) do not change when A depends on $x^0, \ldots, x^3$. If we substitute these expressions into the five dimensional action,

$$S = -\frac{\Lambda}{4} \int d^5x e g^{MP} g^{NQ} F_{MN} F_{PQ} \,, \tag{3.26}$$

we find

$$\begin{aligned} S &= -\frac{\Lambda}{2} \int d^5x \bar{e} e^{-2\sigma} (1 + Ae^{2\sigma})^2 \bar{g}^{mn} F_{m5} F_{n5} \\ &= -\frac{\Lambda}{2\kappa^2} \int d^5x \bar{e} e^{2\sigma} \left[\frac{(1+A)(1+Ae^{2\pi r\Lambda})}{(1+Ae^{2\sigma})(e^{2\pi r\Lambda}-1)} \right]^2 \bar{g}^{mn} \partial_m C \partial_n C \\ &= -\frac{1}{2\bar{\kappa}^2} \int d^4x \bar{e} e^{2\pi r\Lambda} \frac{(1+A)(1+Ae^{2\pi r\Lambda})}{(e^{2\pi r\Lambda}-1)^2} \bar{g}^{mn} \partial_m C \partial_n C \\ &= -\frac{1}{8\bar{\kappa}^2} \int d^4x \bar{e} \sinh^{-2}\left(\frac{B}{\sqrt{6}}\right) \bar{g}^{mn} \partial_m C \partial_n C \,. \end{aligned} \tag{3.27}$$

This is a very suggestive result. Comparing with (3.10), we see that the bosonic part of the effective action takes the usual Kähler form,

$$\begin{aligned} S_{\text{eff}} &= \int d^4x \bar{e} \left[-\frac{1}{2\bar{\kappa}^2} \bar{R} - \frac{1}{8\bar{\kappa}^2} \sinh^{-2}\left(\frac{B}{\sqrt{6}}\right) \bar{g}^{mn} (\partial_m B \partial_n B + \partial_m C \partial_n C) \right] \\ &= \int d^4x \bar{e} \left[-\frac{1}{2\bar{\kappa}^2} \bar{R} - K_{T\bar{T}} \bar{g}^{mn} \partial_m T \partial_n \bar{T} \right] , \end{aligned} \tag{3.28}$$

where $\bar{\kappa} T = (B + \mathrm{i}C)/\sqrt{2}$, and $\bar{T}$ is its complex conjugate. In this expression $K_{T\bar{T}}$ is the Kähler metric

$$K_{T\bar{T}} = \frac{1}{4} \sinh^{-2}\left[\frac{\bar{\kappa}(T+\bar{T})}{2\sqrt{3}}\right] , \tag{3.29}$$

derived from the Kähler potential[4]

$$K(T, \bar{T}) = -\frac{3}{\bar{\kappa}^2} \log\left(1 - e^{-\bar{\kappa}(T+\bar{T})/\sqrt{3}}\right) . \tag{3.30}$$

Under these assumptions, the Kaluza-Klein reduction gives rise to a bosonic effective action with Kähler potential (3.30).

This motivates us to choose an ansatz for B_M that preserves F_{m5}, even for fluctuating A. We take

$$\begin{aligned} B_5 &= \frac{\alpha e^{2\sigma}}{\kappa(1+Ae^{2\sigma})^2} C \\ B_m &= \int_0^{x^5} d|z| \partial_m \left[\frac{\alpha e^{2\sigma}}{\kappa(1+Ae^{2\sigma})^2} \right] C \,, \end{aligned} \tag{3.31}$$

[4]This Kähler potential was also derived in [7].

where, as before, $\sigma = \Lambda|x^5|$. It is a short exercise to show that (3.31) gives the same F_{m5}. The ansatz also induces a nonvanishing F_{mn}, a contribution that can be safely ignored because it is of higher order in the derivative expansion.

Note that our expression for B_m is odd, as required. Also note that it is globally defined, in the sense that the integral could have run from either 0 or πr to x^5 (the difference integrates to zero). The nonvanishing B_m is a tadpole induced by the higher Kaluza-Klein excitations of the graviphoton field.

4. The fermionic effective action

In the previous section we found that the bosonic part of the four dimensional effective action is of Kähler form. In this section we derive the effective action for the fermions. We shall see that it is also of Kähler form, as expected for a supersymmetric theory.

Our starting point is the effective theory of the supergravity multiplet presented in ref. [2]. In that paper, the radion multiplet was set to zero, with $A = C = \Psi_5 = 0$. The zero mode gravitino was found to be

$$\psi_m^+ = \frac{1}{\sqrt{2}}\left(\frac{\bar\kappa}{\kappa}\right) e^{-\sigma/2}\psi_m\,, \qquad \psi_m^- = \frac{1}{\sqrt{2}}\left(\frac{\bar\kappa}{\kappa}\right) e^{-\sigma/2}\mathrm{sgn}(z)\psi_m\,, \tag{4.1}$$

where the four dimensional gravitino ψ_m depends on the coordinates $x^0, \ldots, x^3$.

The four dimensional effective action for the supergravity multiplet was derived by substituting the expressions (3.3) and (4.1) into (2.1) and integrating over x^5. This gives

$$S_{\text{eff}} = \int d^4x\bar e\left[-\frac{1}{2\bar\kappa^2}\bar R + \epsilon^{mnpq}\bar e_n{}^a\overline{\psi}_m\bar\sigma_a D_p\psi_q\right], \tag{4.2}$$

plus four-Fermi terms. Equation (4.2) is the $N = 1$ supergravity action in four dimensions, with an effective four dimensional squared Planck mass $\bar\kappa^{-2}$.

The Randall-Sundrum background preserves one four dimensional supersymmetry. For the case at hand, this supersymmetry is generated by the following Killing spinors,

$$\eta^+ = \frac{1}{\sqrt{2}}e^{-\sigma/2}\eta\,, \qquad \eta^- = \frac{1}{\sqrt{2}}e^{-\sigma/2}\mathrm{sgn}(z)\eta\,, \tag{4.3}$$

where the spinor η is a function of $x^0, \ldots, x^3$, but not x^5. The effective four dimensional supersymmetry transformations can be found by substituting these zero mode expressions into the supersymmetry transformations (2.6). All x^5-dependent terms cancel, leaving

$$\begin{aligned}\delta\bar e_m{}^a &= \mathrm{i}\bar\kappa(\eta\sigma^a\bar\psi_m + \bar\eta\bar\sigma^a\psi_m)\\ \delta\psi_m &= \frac{2}{\bar\kappa}D_m\eta\,,\end{aligned} \tag{4.4}$$

up to three Fermi terms. These are the four dimensional $N = 1$ supersymmetry transformations under which the efffective action is invariant.

We now wish to repeat this exercise in the radion background. We start by computing the five dimensional equations of motion for the fermion fields. We find

$$\begin{aligned}
\bar\sigma^n \hat D_5 \psi_n^+ - \bar\sigma^n \hat D_n \psi_5^+ + \frac{\Lambda}{2} e_{5\hat 5} \bar\sigma^n \psi_n^- + 2\mathrm{i}\Lambda \bar\psi_5^+ - \frac{\kappa}{\sqrt6} g_{55} F^{n5} \bar\psi_n^- - \\
- \frac{\mathrm{i}\kappa}{2\sqrt6} e_{\hat 5 5} F^{n5} \bar\sigma_n \psi_5^+ - \kappa \sqrt{\frac{2}{3}} F_{m5} \bar\sigma^{mn} \bar\psi_n^- &= 0 \\
\bar\sigma^n \hat D_5 \psi_n^- - \bar\sigma^n \hat D_n \psi_5^- + \frac{\Lambda}{2} e_{5\hat 5} \bar\sigma^n \psi_n^+ - 2\mathrm{i}\Lambda \bar\psi_5^- + \frac{\kappa}{\sqrt6} g_{55} F^{n5} \bar\psi_n^+ - \\
- \frac{\mathrm{i}\kappa}{2\sqrt6} e_{5\hat 5} F^{n5} \bar\sigma_n \psi_5^- - \kappa \sqrt{\frac{2}{3}} F_{m5} \bar\sigma^{mn} \bar\psi_n^+ - 2\bar\sigma^n \psi_n^+ [\delta(z) - \delta(z - \pi r)] &= 0 \\
\sigma^{mn} \hat D_m \psi_n^\pm \pm \frac{3\mathrm{i}\Lambda}{4} \sigma^n \bar\psi_n^\pm - \frac{\mathrm{i}\kappa}{4} \sqrt{\frac{3}{2}} e_{5\hat 5} F^{5n} \psi_n^\pm &= 0\,.
\end{aligned} \tag{4.5}$$

In this expression, all fields are five dimensional, $\sigma^m = \sigma^a e_a{}^m$, and we drop terms that depend on the field strength F_{mn} and the off-diagonal vierbein elements $e_m{}^{\hat 5}$ and $e_5{}^a$. The covariant derivatives are

$$\begin{aligned}
\hat D_5 \psi_n^\pm &= D_5 \psi_n^\pm \mp \frac{\mathrm{i}}{2} \omega_{5a\hat 5} \sigma^a \bar\psi_n^\mp - \mathrm{i}\kappa\Lambda \sqrt{\frac{3}{2}} B_5 \psi_n^\mp \\
\hat D_n \psi_5^\pm &= D_n \psi_5^\pm \mp \frac{\mathrm{i}}{2} \omega_{na\hat 5} \sigma^a \bar\psi_5^\mp - \mathrm{i}\kappa\Lambda \sqrt{\frac{3}{2}} B_n \psi_5^\mp \\
\hat D_m \psi_n^\pm &= D_m \psi_n^\pm \mp \frac{\mathrm{i}}{2} \omega_{ma\hat 5} \sigma^a \bar\psi_n^\mp - \mathrm{i}\kappa\Lambda \sqrt{\frac{3}{2}} B_m \psi_n^\mp .
\end{aligned} \tag{4.6}$$

All spin connections are fully five dimensional.

In the radion background, we take our ansatz to be

$$\begin{aligned}
\psi_n^- &= \mathrm{sgn}(z) \psi_n^+ \\
\psi_n^+ &= \frac{1}{\sqrt2} \left(\frac{\bar\kappa}{\kappa}\right) \left[V W \psi_n - \frac{\mathrm{i}\alpha}{\sqrt6} V^{-5} W \sigma_n \bar\chi \right] \\
\psi_5^- &= \mathrm{sgn}(z) \psi_5^+ + \frac{2\alpha}{\sqrt3} \left(\frac{\bar\kappa}{\kappa}\right) e_{5\hat 5} V^{-5} W^* \chi .
\end{aligned} \tag{4.7}$$

In these expressions,

$$V = e^{-\sigma/2} (1 + A e^{2\sigma})^{1/4} \tag{4.8}$$

is the fermion warp factor and

$$W = \exp\left[\mathrm{i}\kappa\Lambda \sqrt{\frac{3}{2}} \int_0^{x^5} d|z| B_5 \right] \tag{4.9}$$

is a Wilson line along the fifth direction. Our ansatz satisfies the fermionic equations of motion for constant A and C. The Wilson line reproduces the x^5-dependent gauge transformations of the five dimensional fermions; the zero mode fields are gauge invariant. Note that the difference between integrating from 0 or from πr is an x^5-independent phase. This phase can be absorbed by a Kähler transformation in the four dimensional theory.

Let us first derive the χ equation of motion. Subtracting the first two equations in (4.5) and substituting (4.7), we find

$$\begin{aligned}0 = \mathrm{i}\alpha^2 \left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2 \bar{\sigma}^m D_m\chi + \frac{\mathrm{i}}{2}\partial_m \left[\alpha^2\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2\right]\bar{\sigma}^m\chi + \\ + \kappa\alpha^2\sqrt{\frac{3}{2}}\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2\left(\Lambda\int_0^{x^5} d|z|F_{m5} + \frac{1}{6}e^{\hat{5}5}F_{m5}\right)\bar{\sigma}^m\chi + \\ + \frac{1}{2}e^{2\sigma}\alpha\left(\sqrt{\frac{3}{2}}\partial_m e_{5\hat{5}} - \mathrm{i}\kappa F_{m5}\right)\bar{\sigma}^n\sigma^m\bar{\psi}_n\,. \end{aligned} \tag{4.10}$$

In these expressions, we take $\sigma^m = \sigma^a \bar{e}_a{}^m$ and we evaluate the spin connections in the radion background,

$$\begin{aligned}\omega_{nab} &= \bar{\omega}_{nab} + (e_a{}^m e_{nb} - e_b{}^m e_{na})\frac{\partial_m Ae^{2\sigma}}{2(1+Ae^{2\sigma})} \\ \omega_{na\hat{5}} &= \Lambda \mathrm{sgn}(z) e_{na} \\ \omega_{5a\hat{5}} &= -\frac{\partial_m Ae^{2\sigma}}{(1+Ae^{2\sigma})^2} e_a{}^m\,,\end{aligned} \tag{4.11}$$

where $\omega_{5ab} = 0$ and $\bar{\omega}_{mab}$ is the four dimensional spin connection with respect to $\bar{e}_m{}^a$.

Equation (4.10) does not hold point-by-point in x^5, so the ansatz (4.7) is inconsistent. As before, the four dimensional equations of motion can be found by averaging over the fifth dimension. This gives

$$\begin{aligned}0 = \int_0^{\pi r} dz e^{-2\sigma}\Bigg\{\mathrm{i}\alpha^2 \left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2 \bar{\sigma}^m D_m\chi + \frac{\mathrm{i}}{2}\partial_m \left[\alpha^2\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2\right]\bar{\sigma}^m\chi + \\ + \kappa\alpha^2\sqrt{\frac{3}{2}}\left(\frac{e^{2\sigma}}{1+Ae^{2\sigma}}\right)^2\left(\int_0^{\sigma} d\sigma' F_{m5} + \frac{1}{6}e^{\hat{5}5}F_{m5}\right)\bar{\sigma}^m\chi + \\ + \frac{1}{2}e^{2\sigma}\alpha\left(\sqrt{\frac{3}{2}}\partial_m e_{5\hat{5}} - \mathrm{i}\kappa F_{m5}\right)\bar{\sigma}^n\sigma^m\bar{\psi}_n\Bigg\}, \end{aligned} \tag{4.12}$$

which implies

$$0 = \mathrm{i}\bar{\sigma}^m D_m\chi + \frac{\mathrm{i}}{2}(K^{T\bar{T}}\partial_m K_{T\bar{T}})\bar{\sigma}^m\chi + \frac{\bar{\kappa}}{\sqrt{2}}\partial_m\bar{T}\bar{\sigma}^n\sigma^m\bar{\psi}_n \; +$$

$$
\begin{aligned}
&+\frac{1}{6}\sqrt{\frac{3}{2}}\left[\frac{1-(2+A)e^{2\pi r\Lambda}}{1-e^{2\pi r\Lambda}}\right]\partial_m C\bar{\sigma}^m\chi \\
&= \mathrm{i}\bar{\sigma}^m\tilde{D}_m\chi+\frac{\bar{\kappa}}{\sqrt{2}}\partial_m\bar{T}\bar{\sigma}^n\sigma^m\bar{\psi}_n\,. \qquad (4.13)
\end{aligned}
$$

Here

$$
\tilde{D}_m\chi=\partial_m\chi+\frac{1}{2}\bar{\omega}_{mab}\sigma^{ab}\chi+\Gamma^T_{TT}\partial_m T\chi-\frac{\bar{\kappa}^2}{4}\left(K_T\partial_m T-K_{\bar{T}}\partial_m\bar{T}\right)\chi \qquad (4.14)
$$

is the covariant derivative, with Kähler connection

$$
\Gamma^T_{TT}=-\frac{\bar{\kappa}}{\sqrt{3}}\coth\left[\frac{\bar{\kappa}(T+\bar{T})}{2\sqrt{3}}\right]. \qquad (4.15)
$$

Equation (4.13) is the effective four dimensional χ equation of motion.

Let us now derive the four dimensional gravitino equation of motion. Using the third equation in (4.5), together with the previous results, we find

$$
\begin{aligned}
0=\epsilon^{mnpq}&\left[\bar{\sigma}_n D_p\psi_q+\mathrm{i}\kappa\sqrt{\frac{3}{2}}\left(\Lambda\int_0^{x^5}d|z|F_{p5}-\frac{1}{2}e^{\hat{5}5}F_{p5}\right)\bar{\sigma}_n\psi_q\right]- \\
&-\frac{\alpha}{2}e^{2\sigma}\left(\sqrt{\frac{3}{2}}\partial_n e_{5\hat{5}}+\mathrm{i}\kappa F_{n5}\right)\sigma^n\bar{\sigma}^m\bar{\chi} \qquad (4.16)
\end{aligned}
$$

where, as before, $\sigma^m=\sigma^a\bar{e}_a{}^m$. Averaging over the fifth dimension gives

$$
\begin{aligned}
0=\int_0^{\pi r}dz&\left\{e^{-2\sigma}\epsilon^{mnpq}\left[\bar{\sigma}_n D_p\psi_q+\mathrm{i}\kappa\sqrt{\frac{3}{2}}\left(\int_0^{\sigma}d\sigma' F_{p5}-\frac{1}{2}e^{5\hat{5}}F_{p5}\right)\bar{\sigma}_n\psi_p\right]-\right. \\
&\left.-\frac{\alpha}{2}\left(\sqrt{\frac{3}{2}}\partial_n e_{5\hat{5}}+\mathrm{i}\kappa F_{n5}\right)\bar{\sigma}^n\sigma^m\bar{\chi}\right\}. \qquad (4.17)
\end{aligned}
$$

This implies

$$
\begin{aligned}
0&=\epsilon^{mnpq}\bar{\sigma}_n\left[D_p\psi_q-\frac{\mathrm{i}}{2}\sqrt{\frac{3}{2}}\frac{1+Ae^{2\pi r\Lambda}}{e^{2\pi r\Lambda}-1}\partial_p C\psi_q\right]- \\
&\quad-\frac{\alpha}{2}\left(\frac{\bar{\kappa}}{\kappa}\right)^2\partial_n(B+\mathrm{i}C)\bar{\sigma}^n\sigma^m\bar{\chi} \\
&=\epsilon^{mnpq}\bar{\sigma}_n\tilde{D}_p\psi_q-\frac{\bar{\kappa}}{\sqrt{2}}K_{T\bar{T}}\partial_n T\bar{\sigma}^n\sigma^m\bar{\chi}\,, \qquad (4.18)
\end{aligned}
$$

where the covariant derivative is

$$
\tilde{D}_m\psi_n=\partial_m\psi_n+\frac{1}{2}\bar{\omega}_{mab}\sigma^{ab}\psi_n+\frac{\bar{\kappa}^2}{4}\left(K_T\partial_m T-K_{\bar{T}}\partial_m\bar{T}\right)\psi_n\,. \qquad (4.19)
$$

Equation (4.18) is the equation of motion for the gravitino ψ_m.

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The fermionic equations of motion (4.13) and (4.18), together with their bosonic partners (3.12) and (3.13), are precisely the equations of motion that follow from the following four dimensional effective action,

$$S_{\text{eff}} = \int d^4x \bar{e} \Big[-\frac{1}{2\bar{\kappa}^2}\bar{R} + \epsilon^{mnpq}\bar{\psi}_m\bar{\sigma}_n\tilde{D}_p\psi_q - \\ - K_{T\bar{T}}\bar{g}^{mn}\partial_m T\partial_n\bar{T} - \mathrm{i}K_{T\bar{T}}\bar{\chi}\bar{\sigma}_m\tilde{D}_m\chi - \\ - \frac{\bar{\kappa}}{\sqrt{2}}K_{T\bar{T}}\partial_n\bar{T}\chi\sigma^m\bar{\sigma}^n\psi_m - \frac{\bar{\kappa}}{\sqrt{2}}K_{T\bar{T}}\partial_n T\bar{\chi}\bar{\sigma}^m\sigma^n\bar{\psi}_m \Big]. \quad (4.20)$$

In fact, we have carried out the nontrivial cross-check to show that (4.20) can be obtained by substituting (3.7), (3.31) and (4.7) into (2.1) and integrating over x^5.

5. Supersymmetry transformations

In the previous section we found the effective action for the radion supermultiplet. We saw that it was of supersymmetric form, with Kähler potential (3.30). In this section we derive the four dimensional supersymmetry transformations directly from the five dimensional transformations (2.6).

The derivation of the supersymmetry transformations is more subtle than the derivation of the effective action. The action involves an integral over x^5, but the supersymmetry transformations hold point-by-point in x^5. Therefore, to derive the supersymmetry transformations, it is helpful to choose an ansatz in which the radion does not mix with the graviton, at any point in x^5. Our expression (3.7) has precisely this property.

Using our ansatz, it is straightforward to derive the supersymmetry transformation laws. We start with the Killing spinors,

$$\eta^+ = \frac{1}{\sqrt{2}}VW\eta\,, \qquad \eta^- = \frac{1}{\sqrt{2}}VW\mathrm{sgn}(z)\eta \quad (5.1)$$

which parameterize the one unbroken supersymmetry in the radion background. We substitute (3.7), (3.31), (4.7) and (5.1) into (2.6) and cancel all x^5 dependence. In this way we derive the transformation laws

$$\begin{aligned} \delta\bar{e}_m{}^a &= \mathrm{i}\bar{\kappa}(\eta\sigma^a\bar{\psi}_m + \bar{\eta}\bar{\sigma}^a\psi_m) \\ \delta\psi_m &= \frac{2}{\bar{\kappa}}\tilde{D}_m\eta \\ \delta T &= \sqrt{2}(\eta\chi + \bar{\eta}\bar{\chi}) \\ \delta\chi &= \mathrm{i}\sqrt{2}\sigma^m\bar{\eta}\partial_m T\,, \end{aligned} \quad (5.2)$$

under which (4.20) is invariant. In these transformations, the covariant derivative of η is given by

$$\tilde{D}_m\eta = \partial_m\eta + \frac{1}{2}\bar{\omega}_{mab}\sigma^{ab}\eta + \frac{\bar{\kappa}^2}{4}\left(K_T\partial_m T - K_{\bar{T}}\partial_m\bar{T}\right)\eta\,. \quad (5.3)$$

Let us demonstrate how this works for the case of the radion multiplet. (The transformation laws for the supergravity multiplet are found using the same technique.) We first compute the variation of T,

$$
\begin{aligned}
\delta T &= \frac{1}{\sqrt{2}\bar{\kappa}}(\delta B + \mathrm{i}\delta C) \\
&= \frac{\Lambda}{\bar{\kappa}} \int_0^{\pi r} dz(\sqrt{3}\delta e_{5\hat{5}} + \mathrm{i}\sqrt{2}\kappa\delta B_5) \\
&= 2\sqrt{3}\Lambda\left(\frac{\kappa}{\bar{\kappa}}\right)\int_0^{\pi r} dz(\eta^+\psi_5^- - \eta^-\psi_5^+) + \text{h.c.} \\
&= 2\sqrt{2}\alpha\Lambda \int_0^{\pi r} dz V^{-4} e_{5\hat{5}}(\eta\chi + \bar{\eta}\bar{\chi}) \\
&= \sqrt{2}(\eta\chi + \bar{\eta}\bar{\chi})\,,
\end{aligned}
\tag{5.4}
$$

where we drop all three Fermi and higher derivative terms. In a similar way, we compute the variation of χ,

$$
\begin{aligned}
\delta\chi &= \frac{\sqrt{3}}{2\alpha}\left(\frac{\kappa}{\bar{\kappa}}\right) e^{\hat{5}5} V^5 W(\delta\psi_5^- - \mathrm{sgn}(z)\delta\psi_5^+) \\
&= -\frac{\mathrm{i}}{\bar{\kappa}}\sigma^m\bar{\eta}\left(\sqrt{\frac{3}{2}}\frac{1}{\alpha}\partial_m A - \mathrm{i}\partial_m C\right) \\
&= \mathrm{i}\sqrt{2}\sigma^m\bar{\eta}\partial_m T.
\end{aligned}
\tag{5.5}
$$

All x^5 dependence cancels, leaving the supersymmetry transformations of the low energy four dimensional effective theory.

6. Conclusions

In this paper we derived the effective action and supersymmetry transformations for the radion supermultiplet in the supersymmetric Randall-Sundrum scenario with two opposite tension branes. We found the action to be of the standard supersymmetric form, with Kähler potential

$$
K(T,\bar{T}) = -\frac{3}{\bar{\kappa}^2}\log\left(1 - e^{-\bar{\kappa}(T+\bar{T})/\sqrt{3}}\right). \tag{6.1}
$$

The supersymmetric action is the starting point for studies of radion stabilization in supersymmetric theories [7].

The Kähler potential (6.1) has the correct limit as the warp factor vanishes. Indeed, rescaling $T \to \Lambda T$, $\chi \to \Lambda\chi$ and taking $\Lambda \to 0$, it is not hard to see that the effective action reduces to the usual no-scale form, with Kähler potential

$$
K(T,\bar{T}) \to -\frac{3}{\bar{\kappa}^2}\log(T+\bar{T})\,. \tag{6.2}
$$

It is perhaps worth noting that a simple argument suggests the above form for the Kähler potential. The argument proceeds as follows:

1. Start with the four dimensional supergravity action, with four dimensional Planck constant $\bar{\kappa}^2$, where $\bar{\kappa}^{-2} = \kappa^{-2}(1 - e^{-2\pi r\Lambda})$. Then transform to the supergravity frame, in which the Einstein term takes the following form,

$$S_{\rm sugra} = -\frac{1}{2\bar{\kappa}^2}\int d^4x\bar{e}e^{-\bar{\kappa}^2K(T,\bar{T})/3}\bar{R}\,. \tag{6.3}$$

   The Kähler potential $K(T,\bar{T})$ is an unknown real function of the complex radion field T.

2. Note that for constant radion, the Kähler potential is itself a constant, so the action (6.3) describes Einstein gravity with an effective Planck constant $\bar{\kappa}^{-2}\exp(-\bar{\kappa}^2K/3)$. The Kähler potential must vanish when T is set to its expectation value, $\langle T+\bar{T}\rangle = 2\sqrt{3}\pi r\Lambda/\bar{\kappa}$.

3. Now shift the radion expectation value, $\langle T+\bar{T}\rangle \to 2\sqrt{3}\pi r'\Lambda/\bar{\kappa}$. According to (6.3), this changes the effective four dimensional Planck constant from $\bar{\kappa}^{-2}$ to $\bar{\kappa}'^{-2} = \bar{\kappa}^{-2}\exp(-\bar{\kappa}^2K(T',\bar{T}')/3)$. Consistency then requires $\bar{\kappa}'^{-2} = \kappa^{-2}(1 - e^{-2\pi r'\Lambda})$.

4. Combine these results to find

$$K(T,\bar{T}) = -\frac{3}{\bar{\kappa}^2}\log\left[\frac{1-e^{-\bar{\kappa}(T+\bar{T})/\sqrt{3}}}{1-e^{-2\pi r\Lambda}}\right]. \tag{6.4}$$

   Up to a constant, this is precisely the Kähler potential (6.1).

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